Advanced approaches in PDS/POT modelling of extreme hydrological events

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INTRODUCTION

The partial duration series (PDS) method, as an alternative to the annual maximum series (AMS) method, for analysing extreme hydrological events was introduced through a number of early papers (Langbein, 1949; Borgman, 1963; Bernier, 1967; Shane and Lynn, 1964; Todorovic and Zelenhasic, 1970; Cunnane, 1973; Rosbjerg, 1977). The method considers exceedances of a pre-selected threshold, and the basic assumptions comprise Poisson arrival rates and exponentially distributed exceedance magnitudes (Zelenhasic (1970) also considered gamma distributed exceedances). Although the method can handle all kinds of extremes, it has been applied primarily to flood studies, and has for that reason also been denoted the peaks over threshold (POT) method.

In the 1980s and early 1990s the method was generalised in different ways, including time-dependent parameters (North, 1980), correlated peak values (Rosbjerg, 1985; 1987a), risk estimation (Konecny and Nachtnebel, 1985; Rasmussen and Rosbjerg, 1989), Bayesian approaches (Rousselle and Hindie, 1976; Rasmussen and Rosbjerg, 1991a), a fixed number of peaks (Buishand, 1989), seasonality (Rasmussen and Rosbjerg, 1991b), and alternatives to the exponential distribution of the threshold exceedances comprising Weibull (Miguel, 1984; Ekanayake and Cruise, 1993), log-normal (Rosbjerg, 1987b; Rosbjerg *et al*., 1991), and generalised Pareto (Van Montfort and Witter, 1986; Hosking and Wallis, 1987; Fitzgerald, 1989; Davison and Smith, 1990; Wang, 1991; Rosbjerg *et al*., 1992).

The purpose of this paper is to summarise and present some important extensions of the PDS/POT method since the mid 1990s. Selected papers will be briefly introduced and the main obtained results referenced. Finally, a conclusion on the future applicability of the method is given.

COMPARISON OF AMS AND PDS METHODS FOR AT-SITE ESTIMATION

The PDS method with exponentially (EXP) distributed exceedances corresponds to the Gumbel or extreme value type 1 (EV1) distribution for the AMS. Cunnane (1973) compared the AMS/EV1 and PDS/EXP using maximum likelihood (ML) estimation and showed that the PDS approach should be preferred, if the average number of peak exceedances per year in large samples was greater that 1.65. This result was generalised by Madsen *et al*. (1997a) by using the generalised extreme value distribution (GEV) in AMS and, correspondingly, the generalised Pareto (GP) distribution in PDS and by comparing different estimation methods. A more limited approach was taken by Wang (1991), who compared AMS/GEV and PDS/GP in which the sample size was equal to the number of sampled years, and considered only estimation by the method of probability weighted moments (PWM).

Madsen *et al.* (1997a) found that PDS/GP in most cases will outperform AMS/GEV. In the case of ML estimation, the PDS/GP provides the most efficient T-year event estimation. Using either method of moment (MOM) or PWM estimation, PDS/GP is generally preferable for negative shape parameters. The overall assessment was that for negative shape parameters PDS/GP should be applied using MOM estimation, for shape parameters around zero, PDS/EXP should be used, for moderately positive shape parameters AMS/GEV with MOM estimation should be applied, and for strong positive shape parameters PDS/GP should be applied with ML estimation. Since positive shape parameters are rarely found in hydrology, PDS/GP or PDS/EXP are to be preferred in comparison with AMS/GEV or AMS/EV1.

EXTENSIONS TO REGIONAL APPROACHES

Index-flood approach

The general objective in regional methods is to obtain more efficient quantile estimates at a particular site by combining information form different gauging stations in a region that can be assumed to have similar hydrological behaviour. Madsen and Rosbjerg (1997a) introduced a regional index-flood method based on PDS/GP. The index-flood method prescribes that the extremes events in the selected region are identical except for scale. A basic problem is that strictly homogeneous regions do not exist and, hence, a trade-off must be found between the degree of heterogeneity and the value of including additional information. It was found that for small to moderate sample sizes, the regional estimator is superior to the at-site estimator, even in extremely heterogeneous regions, the performance of the regional estimator being relatively better in regions with a negative shape parameter. When the record length increases, the relative performance of the regional estimator decreases, however, still being preferable in comparison to the at-site estimator for large sample sizes in moderately heterogeneous regions. Modest intersite dependence has only a small effect on the performance of the regional index-flood estimator.

Comparison between AMS and PDS methods

As for at-site estimation, corresponding regional index-flood approaches can be obtained using the PDS/GP and AMS/GEV models. Madsen *et al.* (1997b) accomplished a comprehensive comparison of the two cases based on, respectively, Monte Carlo simulations and a case study from New Zealand. In typical regions with a realistic degree of heterogeneity, the Monte Carlo simulations showed that the PDS/GP index-flood model is more efficient than the AMS/GEV model. The case study involved 48 catchments from the South Island of New Zealand. To identify approximately homogeneous groupings of catchments, a split-sample regionalisation approach based on catchment characteristics was adopted. It was found that the defined groups are more homogeneous for PSD data than for AMS data (a two-way grouping based on annual average rainfall (AAR) is sufficient for PDS, while for AMS a further partitioning is necessary). Another and very interesting experience is that in the determination of the regional parent distribution based on L-moment diagrams, PDS data, in contrast to AMS data, provide an unambiguous interpretation supporting GP.

Regional Generalized Least Squares (GLS) and Empirical Bayesian Estimation

In the last paper of a series of four, Madsen and Rosbjerg (1997b) combined the PDS/GP index-flood concept with an empirical Bayes procedure. The prior regional information is inferred through GLS regression that explicitly accounts for intersite correlation and sampling uncertainties of the PDS parameters. Two different Bayesian T-year event estimators were considered: a linear estimator that requires only some moments of the prior distributions to be specified and a parametric estimator that is based on specified families of prior distributions. The New Zealand data were again used for illustration. It was found that in the case of a strongly heterogeneous intersite correlation structure, the GLS procedure provides a more efficient estimate of the regional GP shape parameter as compared to the usually applied weighted regional average. If intersite dependence is ignored, the uncertainty of the regional estimator may be seriously underestimated and erroneous conclusions with respect to regional homogeneity may be taken. The GLS procedure was found to provide a general framework for a reliable evaluation of parameter uncertainty as well as for an objective appraisal of regional homogeneity. A comparison of the two different Bayesian T-year event estimators revealed that in most cases the simple linear estimator is adequate.

Regional estimation of intensity-duration-frequency (IDF) curves

Based on the above regional approach, Madsen *et al.* (2002) developed a general framework for regional analysis and modelling of extreme rainfall characteristics. In the PDS model, the average annual number of threshold exceedances, the mean value of the exceedance magnitudes, and the L-coefficient of variation are considered as regional variables. A GLS regression model was applied for evaluating the regional heterogeneity of the PDS parameters. For the parameters that show a significant regional variability, the GLS mode was subsequently adopted for describing the variability from physiographic and climatic characteristics and, for determination of a proper regional parent distribution, L-moment analysis was applied for discriminating between the exponential distribution and various two-parameter distributions in the PDS model. The analyses revealed that the GP distribution should be used as a regional parent for exceedance magnitudes, and the mean annual precipitation as explanatory variable for the average annual number of exceedances. The resulting model can be used for estimation of rainfall IDF curves at arbitrary locations in the region considered, including realistic uncertainty bounds. The model has been successfully implemented in Denmark for provision of design rainfall characteristics.

OPERATIONAL GUIDELINES

Lang *et al*. (1999) reviewed the PDS method in order to provide recommendations for its operational use. Initially, they studied the problem of threshold selection and recommended use of a set of comparative tests. First, an interval for acceptable thresholds should be found by a test for stability of the distribution parameters and a test for verification of Poissonian occurrences. Secondly, within the identified interval, the threshold should be selected as the largest one corresponding to at least an average number of exeedances of 2 (or 3) per year. The occurrence process of threshold exceedances was studied in detail, and they recommended using different statistical tests to verify the basic assumptions: a dispersion index test, a stationarity test and a seasonality test. For modelling threshold exceedances in particular the exponential and the generalised Pareto distributions were considered. A complementary AMS analysis was recommended always to be carried out due its relative simplicity, and to allow for comparisons.

GENERALISED MAXIMUM LIKELIHOOD ESTIMATORS

Martins and Stedinger (2000) introduced a generalised maximum likelihood (GML) quantile estimator for the GEV in AMS analysis and showed that it performed substantially better than MOM and L-moment quantile estimators for values of the shape parameter of interest. The ML estimator is generalised by using a Bayesian prior distribution (a Beta distribution) to restrict values of the shape parameter to a statistically/physically reasonable range. Subsequently, Martins and Stedinger (2001) applied the same principle in PDS analysis. It was found that for negative values of the shape parameter (the range of primary interest), PDS/GP and AMS/GEV have approximately the same performance. Moreover, the result was found to be insensitive to the mean annual number of threshold exceedances. The paper further demonstrated that the approximate PDS/EXP performs better than the true PDS/GP model only in a narrow range of shape parameter values around zero, decreasing for increasing record length. The conclusion that the advantage of using PDS in comparison to AMS is vanishing when using GML estimation is surprising, taking the strong support for PDS analysis obtained by Madsen *et al*. (1997a) into account. Possibly the discrepancy can be explained by the different Monte Carlo simulation procedures used in the two studies. By using a prior to restrict the shape parameter, the uncertainty of the shape parameter estimate can be reduced significantly. This uncertainty is the main contributor to the uncertainty of the T-year event estimate and, since PDS and AMS use the same prior, it may reduce the difference between the two estimates. Generally, care should be taken using a too strong prior.

FILTERING OF CONTINOUS STREAMFLOW DATA

Claps and Laio (2003) reconsidered the classical PDS approach and recommended a new procedure for threshold selection, based on filtering of the continuous (or daily) streamflow data and denoted the filtered peak over threshold (FPOT) method. Their method leads to much larger values for the average annual number of extreme events than those obtained by more usual threshold selection procedures. To address potential problems due to selection of more extreme events, they presented a comprehensive set of tests for proving that the basic PDS assumptions can be retained when using the FPOT approach. This includes test for independence between successive peaks, test for the distribution of peak occurrences, and test for GP distribution of peak magnitudes. A discussion of efficiency in T-year event estimation led to the choice of the Bayesian method for deriving the distribution of the T-year event. The accuracy was subsequently determined by 90% credible limits. By applying the FPOT method to a large number of daily runoff series, they were able, through reduction of the credible intervals, to demonstrate the advantage of selecting larger samples than is usually done in PDS analysis.

BAYSIAN METHODS FOR INCORPOTING HISTORICAL DATA AND EXPERT JUDGEMENTS

Inclusion of historical data

In a comprehensive study, Parent and Bernier (2003a) presented an at-site Bayesian PDS/GP model suited for inclusion of imprecise historical data. A flexible semi-conjugate prior structure was selected for mathematical convenience and Gibbs sampling introduced to complement the historical record. The poor quality of the historical data was described in a parameter-parsimonious way. Also, a cost function was introduced and used to find a design value that minimises the expected predictive cost. The study showed a clear advantage of the Bayesian framework for incorporation of imprecise historical data. A case study in the Garonne River, where a systematic record of data spanning the period 1913–70 was supplemented with (imprecise) information about 12 main historical floods during the period 1770–1912, resulted in 40% reduction of the 90% credible interval for the T-year event.

Encoding of prior expert judgements

The Bayesian framework is also suitable for systematic incorporation of subjective prior information. This was demonstrated by Parent and Bernier (2003b), who found that judgements of an expert could significantly reduce the uncertainty of design values. In the Garonne River case study, the expert provided prior information (median values and 90% quantiles) of the mean annual number of floods above Q=2500 $m^3\ s^{-1}$, the quantile difference $Q_{100} - Q_{10}$, and the quantile difference ratio $(Q_{1000} - Q_{100})/(Q_{100} - Q_{10})$. The posterior analysis was based on direct Monte Carlo sampling, and it was found that use of the informative prior in comparison to a non-informative prior resulted in a reduction of the credible interval of Q_{100} of the same order of magnitude as obtained by inclusion of historical data.

CONCLUSIONS

In recent years a number of different contributions to the PDS model has substantially advanced the applicability of the model for assessment of extreme hydrological events. It has been demonstrated that the PDS/GP model is competitive with the AMS/GEV model and highly efficient for regionalisation. New procedures for testing the basic assumptions have been developed, generalised maximum likelihood introduced and filtering methods developed for selection of independent threshold exceedances. Finally, the strengths of Bayesian methods in PDS analysis have been rigorously demonstrated. The method is found attractive for analysis of extreme hydrological events.

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