A multiobjective optimisation model for regional water resources allocation for sustainable development

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INTRODUCTION

Water resources shortage is a global problem, particularly in the developing countries because of rapid population growth and water pollution due to uncontrolled economic development. In some regions it is difficult to allocate as much water to all users as they would like, and the rational allocation of water resources is a key issue. This paper presents a multiobjective optimal water resources allocation model from the viewpoint of sustainable development. This model is employed to allocate the water resources of Longmen County in southern China where water shortage cannot be avoided due to the rapid growth in water demand. The result shows that the water allocation scheme is reasonable and acceptable.

THE MULTIOBJECTIVE OPTIMAL WATER RESOURCES MODEL

Optimal water allocation criteria

“Sustainable development is development that meets the need of the present without compromising the ability of future generations to meet their needs” (Brundtland *et al*., 1987) is widely accepted as the criterion for sustainable development. Based on this criterion, two criteria for optimal water resources allocation are derived as satisfying the current water resources demands and maintaining the eco-system integrity. These two criteria can be explained as follows:

Satisfying the current water resources demands: The sustainable development should first satisfy the need for present development; the need in water resources allocation is the need for water resources, so satisfying the current water resources demand is the first criterion for optimal water resources allocation. Satisfying water demands includes supplying water for all current uses, such as agriculture, industry and potable water supplies, all of which can be classified as economic users because water is critical in maintaining normal production, or is of direct benefit to these users. Such benefits can be calculated, based on the quantity of water supplied.

Maintaining eco-system integrity: maintaining ecosystem integrity is to protect natural resources for the needs of future generations. Currently, particularly in the developing countries, development tends to scar the environment, and is not sustainable. For optimal water allocation, maintaining eco-system integrity means providing sufficient water for ecosystem and environment improvement. This is a special water use, and it is for a public user. There is no direct benefit in such water supply, it is a benefit that cannot be calculated in monetary terms, yet water supply for ecosystems must be satisfied, otherwise they deteriorate and the natural resources cycling cannot be maintained, thus compromising the needs for the future generation.

Objective functions

The objective functions of the model can be derived from the optimal water allocation criteria. In selecting an objective function, two principles are applied:

(1) The objective function should be computable, this means a definite value can be calculated from the objective function;

(2) To have as few objective functions as possible so as to simplify the model solution procedure.

Based on these principles, two objective functions are selected, one is to maximise the economic benefits and the other is to maximise the water supply for eco-system users.

Maximising the economic benefits: the economic benefits include the benefits of water supply to all economic users, such as agriculture , industry, the general public and other possible users. The function can be written as:

$$MAX \sum_{j=1}^{m} c_j x_j \tag{1}$$

where m is the number of water users in the region, x_j is water quantity provided to water user j, while c_j is the benefit of one unit water supply for water user j. If c_j is a constant, then the function is a linear function.

Maximising ecosystem water supply: this function is to supply water to ecosystems as much as possible, so the function can be written as:

$$MAX \quad q \tag{2}$$

where q is water supply to ecosystem user; if there are more than one ecosystem users, then q is the sum of all ecosystem users water supply in the region.

Constraints

The following constraints are included in the model.

(1) Water Resources constraints

$$\sum_{j=1}^{m} x_j + q \le R \tag{3}$$

where R is the total water resources available in the whole region.

(2) Water quantity constraints

$$x_j \le xm_j \qquad j = 1,2,......,m \tag{4}$$

where xm_j is the maximum water demands of user j.

(3) Fairness constraints

$$x_j \ge w_j xm_j \qquad j = 1,2,......,m \tag{5}$$

where w_j is the percentage that the water demand to be secured for user j, this constraint is to guarantee that the low-profit users can get the minimum water supply to maintain the basic activities, so to avoid all the water resources to be allocated to the high-profit users.

Water resources allocation model

Combining the objective functions and constraints, the water resources optimal allocation model can be written as following.

$$\begin{cases} MAX \ \sum_{j=1}^{m} c_j x_j \\ MAX \quad q \\ s.t. \quad \sum_{j=1}^{m} x_j \le R \\ \quad x_j \le xm_j \qquad j = 1,2,......,m \\ \quad x_j \ge w_j xm_j \qquad j = 1,2,...,m \end{cases} \tag{6}$$

Model solution

This model is a multiobjective optimisation model with two objective functions; a solving procedure is presented to find the optimal solution of the model. The main goal is to change the second objective function of the multiobjective model into a constraint, thus transfering the multiobjective model into a number of single objective models. By solving the single objective model, the non-inferior solution set can be derived. On this basis, a preference coefficient is derived to choose the optimal solution of the multiobjective model. The method is presented below.

Model transfer

Changing the second objective function into a constraint as following:

$$q = qs_i \tag{7}$$

where qs_i is the water resources allocated to ecosystem user that takes n values between $qs_{\max}$ and $qs_{\min}$; $qs_{\max}$ is the maximum eco-system water demand, while $qs_{\min}$ is the minimum water demand, which can be calculated as:

$$qs_{\min} = w \bullet qs_{\max} \tag{8}$$

Where w is a coefficient. After this treatment, the multiobjective model becomes a single objective model, as follows:

$$\begin{cases} MAX \ \sum_{j=1}^{m} c_j x_j \\ s.t. \quad \sum_{j=1}^{m} x_j + q \le R \\ \quad x_j \le xm_j \qquad j = 1,2,...,m \\ \quad q = qs_i \\ \quad x_j \ge w_j xm_j \qquad j = 1,2,...,m \end{cases} \tag{9}$$

As qs takes n values, so there will be n single objective models which are linear optimisation models, and the Linear Programming(LP) is employed to find the optimal solution of this model.

Optimal solution

The optimal solution of the multiobjective model will be chosen from the non-inferior solutions; this could be an interactive procedure, where the decision-maker will compare every non-inferior solution and choose the optimal solution by referring to one or more preference indices, such as to provide as much water as possible for ecossystem users, to get more direct profit, and to satisfy the water demands for daily life.

CASE STUDY

Longmen County in southern China is taken as the study case. Longmen County is a sub-district of Huizhou City which has long been the political, economic, military, cultural and commercial centre of East Guangdong Province, with a population of 266.53 million and an area of 11 173 km^2, while Longmen County alone has an area of 2267 km^2. According to future projections, water shortage in the year 2010 and beyond is obvious, and not all the demands can be satisfied. The model presented in this paper is used to allocate the water resources of Longmen County.

Available water resources and water demands

According to the projection, available water resources in Longmen County in 2010 is 30 979 m^3, while the total water demand is 38 278 m^3. The users are categorised into five types, including Urban Area Daily Life user (UADL), Rural Area Daily Life user (RADL), Industrial user (IND), Irrigation user (IRR) and ecosystem user (ECO). The projected water demands for these users are listed in Table 1.

From these results, if no water transferred from neighbouring areas, then the water supply warranty ratio in 2010 is only 80.93%, and the water shortage is very serious. Allocating water resources rationallyto every user is a key issue for the regional sustainable development.

Table 1 Projected water demands (unit =10000 m$^{3)}$

year	Total	UADL	IND	IRR	RADL	ECO
2010	38278	1380	1843	32367	1938	950

Multiobjective water allocation model

The multiobjective water allocation model is based on Equation (6) and the values of coefficients c_j and w_j are listed in Table 2.

Table 2. Coefficients of the multiobjective water allocation model (unit = yuan/m$^{3)}$

	UADL	*IND*	*IRR*	*RADL*	*ECO*
c	0.88	385	3.9	0.44	
w	0.9	0	0.80	0.9	0.75

The value of *c* for UADL and RADL is the water supply price in the year 2002, while value for IND and IRR are derived from the value of the units of water consumption for industrial and agricultural production.

Model solution

Based on the solving procedure presented in this paper, the single models are set up; here *qs* takes 10 values between 680 to 950, so 10 single objective models are set up, and by employing linear programming, the solutions for the 10 single objective models are derived. The water allocation schemes for the ten models are listed in Table 3.

From the results of Table 3, it has been found that water supply to UADL, IRR and RADL are the same, while the water supply to IND increases as the ecosystem user's water supply decreases.

In this paper, scheme 4 is chosen as the optimal solution of the multiobjective model because the water supply warranty ratio of ecosystem user reaches 81.05%, while the decreasing water supply to other users does not cause much reduction in the whole direct benefit, but still maintains the water supply warranty ratio to IND above 70%; other schemes cannot have a better trade-off than between ecosystem and IND users.

The water allocation results are shown in Table 4, which shows that the total water supply warranty ratio is 80.93%, while the ratios for ECO, UADL, RADL, IND and IRR are 81.05%, 90%, 90%, 72.11% and 80%, respectively.

Table 3. water allocation schemes for single objective models

Scheme	*ECO*	*UADL*	*IND*	*IRR*	*RADL*
1	680	1242	1419	25894	1744
2	710	1242	1389	25540	1744
3	740	1242	1359	25520	1744
4	770	1242	1329	25500	1744
5	800	1242	1299	25480	1744
6	830	1242	1269	25460	1744
7	860	1242	1239	25440	1744
8	890	1242	1209	25420	1744
9	920	1242	1179	25400	1744
10	950	1242	1149	25380	1744

Table 4. Water resources allocation result

User	*Total*	*UADL*	*IND*	*IRR*	*RADL*	*ECO*
Demand	38278	1380	1843	32367	1938	950
Allocation	30979	1242	1329	25500	1744	770
Warranty ratio	80.93%	90%	72.11%	80%	90%	81.05%

CONCLUSION

This paper presented a multiobjective optimisation model for rationally allocating water resources on a region from the viewpoint of sustainable development. Longmen County in southern China is taken as the study case and reasonable results were derived.

REFERENCES

Bruntland, G. (ed.), 1987. *Our common future*. The World Commission on Environment and Development, Oxford University Press.

Huizhou City water resources bureau, 2001. Comprehensive river planning of Huizhou City. *Report no. GD-002.*

Loucks, D.P.1995. Developing and implementing decision support systems: a critique and a challenge. *Water Resour. Bull*., **31**, 571–5820

Mahammad, B. and Marino, M. A. 1984. Reservoir operation: choice of objective function. *J. Water Resour. Plan. Manage*.,**110**(1).

Feng, S. 1990. T*heory, method and application of multi-objective decision*. Wuhan, Huazhong University of Science and Technology Press.